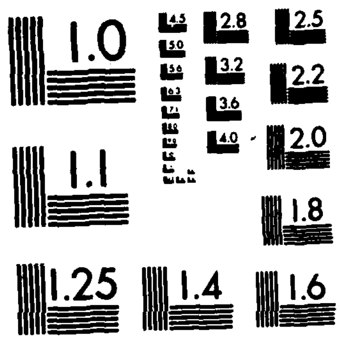
AD-A138 137 A GOAL FOCUSING APPROACH TO ANALYSIS OF INTERGENERATIONAL TRANSFERS OF IN..(U) TEXAS UNIV AT AUSTIN CENTER FOR CYBERNETIC STUDIES A CHARNES ET AL. 1/1

UNCLASSIFIED AUG 83 CCS-RR-466 N00014-82-K-0295 F/G 5/3 NL

END
FILMED
3-84
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Research Report CCS 466

A GOAL FOCUSING APPROACH TO ANALYSIS OF INTERGENERATIONAL TRANSFERS OF INCOME: THEORETICAL DEVELOPMENT AND PRELIMINARY RESULTS

by

A. Charnes
W. W. Cooper
J. J. Rousseau
A. Schinnar**
N. E. Terleckyj***

CENTER FOR CYBERNETIC STUDIES

The University of Texas
Austin, Texas 78712

(11)

Research Report CCS 466

A GOAL FOCUSING APPROACH TO ANALYSIS OF INTERGENERATIONAL TRANSFERS OF INCOME: THEORETICAL DEVELOPMENT AND PRELIMINARY RESULTS

by

A. Charnes
W. W. Cooper
J. J. Rousseau
A. Schinnar**
N. E. Terleckyj***

August 1983

DTIC
ELECTE
FEB 21 1984
A

*Southern Methodist University
**University of Pennsylvania
***National Planning Association

This research was partly supported by ONR Contract N00014-82-K-0295 with the Center for Cybernetic Studies, The University of Texas at Austin; Southern Methodist University; University of Pennsylvania; and the National Planning Association. Reproduction in whole or in part is permitted for any purpose of the United States Government.

CENTER FOR CYBERNETIC STUDIES

A. Charnes, Director
Graduate School of Business 4.138
The University of Texas at Austin
Austin, Texas 78712
(512) 471-1821

ABSTRACT

Existing analytical models of intergenerational transfers of income developed by economists deal with the criterion of equity of retirement systems but not with their stability. These models frequently assume a steady growth in population. However, present and projected population dynamics are characterized by very large changes in the age structure of the population. Governmental and private projection models used for analysis of government policies for social security and other age-determined transfer programs do rely on the current population projections, but they assume independence of future economic growth from the retirement system.

Moreover, none of these models are formulated in a way that makes them operational for studying the numerous dynamic trade-offs and relative possibilities inherent in income transfer programs, for example, the trade-off between taxes and subsidies, between participation rates and income for different groups over time, and the possible effects of cross-subsidization among groups of different compositions and sizes. Such information is central in striving to attain simultaneously the socially desirable goals of maintaining or improving the standards of living of both the working and retired populations and of maintaining an equitable and stable intergenerational transfer system.

This paper proposes such an operationally implementable method of analysis employing the new approach of goal focusing, derived from ideas of goal programming and efficient point analysis. A basic theoretical model is developed and illustrated by numerical example, and extensions in several directions are described. Such models will provide considerable assistance in obtaining substantive knowledge of the prospective magnitudes of the trade-offs involved and their behavior within a set of plausible trends over the next ten to thirty years.

Introduction

The policy decision which will have fundamental implications for the economic and social stability of the United States well into the next century concerns the balancing of three desirable social goals

(1) maintaining or improving the standards of living of the working population;

(2) maintaining or improving the standards of living of the retired population; and

(3) maintaining a stable inter-generational transfer system which permits members of successive generations to be supported in retirement by their successors in the work force.

Standards of living are measured over time by the per capita disposable income for the respective groups. The principal indicator for the functioning of the inter-generational transfer system is the maintenance of a non-negative rate of return between the real payments made to the transfer system in the aggregate by the members of each generation and the payments received by them later.

Historically, transfer payments, and in particular transfer payments to the aged, have been the most rapidly growing component of the government budget expenditures and personal income receipts. In the federal budget, these transfer payments include pensions of government employees, military pensions and the social security programs including disability and medicare benefits. The largest and the most fundamental component of government expenditure, and the one with the most universal ramifications, is the social security program.

The importance of social security as a basic program for income support in the United States is likely to increase rather than to decline in the

future, because the real value of private pension funds has been eroded by slowdown in the economy and by inflation. Also, many companies find themselves unable to expand or even to continue their pension programs and some even may be unable to cover the unfunded pension liabilities from their future profits.

The problem of transfer payments and especially of social security benefits is that the current and anticipated growth rates in the economy do not permit funding of increases in transfer payments at the rate at which they have been growing in the past. It may not even be feasible to provide the future retirees the combination of early retirement age and increases in entitlements which have been built into the existing retirement program without reducing the real disposable income of active workers at the same time. The expected economic growth rates would certainly not support the introduction of new benefit programs, comparable to the past additions of the disability and medicare provisions or automatic cost-of-living adjustments, without imposing large strain on the disposable income of the working population. The need to balance the future incomes of workers and of retirees is the prime policy need. However, the conditions for achieving this balance are not well understood.

Trade-offs between the standards of living of the two populations at a point in time could be established from data on the levels of transfers of real purchasing power from workers to retirees, the size of the two populations and their average disposable incomes. However, such data are not available at present and can only be approximated with some effort. Moreover, the dynamic trade-offs among these goals are much more important than trade-offs at a point in time. They are much more difficult to understand because they are embedded in the processes of economic growth, and they depend on

individual choices between work and leisure which presumably depend on the opportunities for both available to persons at different ages. They involve the issue of preservation of the inter-generational transfer system and require a three-way analysis among these goals. These dynamic trade-offs have not been previously analyzed.

The social significance of this topic is heightened by recent demographic changes. The decline in the birth rate over the past ten years in the United States, which also occurred in other industrialized nations, has changed the population age structure in a way which may raise problems of social and economic stability. Specifically, as the proportion of families with children has declined, participation by women in the labor force has increased rapidly at the same time as the large post-World War II age cohort has been entering the labor force. The resulting growth of the labor force permitted continuation of economic growth in spite of diminished growth in productivity. A relatively high growth in the labor force and employment may continue into the next decade. But, this growth will continue to slow down and, barring massive immigration of workers to the United States, may virtually stop at the end of the century as the bulge of the population will approach retirement age (see Table 1).

The more immediate and pressing problem, however, is the imbalance in the growth rates in disposable real income of the two groups. The average growth rate in real transfer incomes has tended to exceed the growth in earnings since 1960, and in the 1970's the growth in real earnings of the working population even before taxes has been at a virtual standstill while the growth in the average transfer payment has accelerated (see Table 2).

As the number of the retirees and other beneficiaries of the public transfer system grows, so does the burden of taxes placed on the working

TABLE 1

SELECTED INDICATORS OF CHANGES IN THE AGE STRUCTURE OF POPULATION
UNITED STATES, ACTUAL AND PROJECTED, 1960-1995

Period	Persons Age Indicated Annual Average, Thousands			Average Annual Net Change in the Labor Force
	17	30	65	
1960-64	2,992	2,256	1,350	1,020
1965-69	3,577	2,337	1,427	1,740
1070-74	4,014	2,812	1,629	1,780
1975-79	4,213	3,439	1,826	1,570p
1980-84	3,865	3,910	2,045	1,010p
1985-89	3,555	4,322	2,108	800p
1990-94	3,199	4,337	2,023	n.a.
1995-99	3,617	3,803	1,958	n.a.

p. -preliminary NPA projections
n.a.-not available

Sources: U.S. Bureau of the Census, Current Population Reports, Series P-25, Washington, D.C.: U.S. Government Printing Office:
1960-69: No. 519, "Estimates of the Population of the United States, by Age, Sex, and Race: April 1, 1960 to July 1, 1973," April 1974;
1970-77: No. 721, "Estimates of the Population of the United States by Age, Sex, and Race: 1970-to 1977," April 1978;
1978-99: No. 704, "Projections of the Population of the United States: 1977-2050," July 1977; and the National Planning Association.

TABLE 2

SELECTED INDICATORS OF REAL INCOME AND OF CHANGES IN REAL INCOME OF RETIRED AND WORKING POPULATIONS, UNITED STATES, 1940-1976

Period	Federal Pension Beneficiaries[1]	State and Local Pension Beneficiaries	Social Security OASDI Beneficiaries	Average Gross Weekly Earnings of Production Workers[2]	Pre-tax Earnings Per Person Employed[3]
		A. Real Income per Recipient in 1972 Dollars			
1940	$4,101	$2,669	$ —	$ —	$4,721
1950	2,857	1,915	817	92.33	5,952
1960	2,605	2,158	1,102	113.96	7,673
1970	3,715	2,797	1,316	129.10	9,262
1976	4,755	3,142	1,742	128.93	9,596
		B. Annual Rates of Change in Real Income (percent)			
1940-50	-3.7%	3.4%	-	-	2.3%
1950-60	-0.9	1.2	3.0%	2.1%	2.6
1960-70	3.6	2.4	1.8	1.3	1.9
1970-76	4.2	2.3	4.7	0.0	0.6

[1]Includes Federal government civilian and military pensions.
[2]Total private nonagricultural.
[3]Total earnings in the economy and income of proprietors as given in the national income accounts deflated by the consumer price index and divided by total employment including armed forces.

Sources: Alicia H. Munnell, "Pensions for Public Employees," National Planning Association, December 1978, unpublished; Economic Report of the President, 1979; and Department of Labor, Bureau of Labor Statistics.

population (and factors of production generally) quite apart from the growth in the average transfer benefit received. As a result of these economic and demographic trends, barring an unprecedented and improbable upsurge in productivity, the issue of trade-offs between the incomes of the working population and incomes of the retired population, which has already been felt politically, is likely to become progressively more acute. However, no substantive knowledge exists regarding the prospective magnitudes of these trade-offs or their behavior within a given set of plausible trends over the next 10 to 30 years.

This paper seeks to address this deficiency by developing a systematic and operationally implementable method for the analysis of inter-generational transfers of income. To accomplish this we will employ an approach referred to as "goal focusing," a new approach to assessing complex relationships and problems.

The ideas of goal focusing derive in part from those of goal programming from management science and operations research [2]. However, the usual goal programming approach is here joined to the concepts of efficient point analyses and optimality as formulated in economics [3]. That is, in goal focusing one seeks the closest "efficient point" instead of only the "closest point" to the specified goals.

In doing so, it also supplies a variety of other conveniences. Its linear programming equivalents afford computational efficiency and an easy access to sensitivity analyses, etc. The approach also extends to constraint adjunctions, other kinds of model alterations, and other such devices for exploration that may appear to be appropriate.

If desired, these goal focus ideas can be extended to goal intervals when ranges of possibilities are to be considered rather than specific levels or targets [2].

In the following sections we present our basic theoretical development and the results of an illustrative example. Extensions and modifications of our model, already underway, are discussed in a concluding section.

The Basic Model

Consider an economy in which the population is grouped demographically by, for example, age, income, etc., and in which tax payments are made and tax subsidies received by these groups. Let

$p(t)$ = vector of demographic (population) groups in period t,

$a(t)$ = vector of tax-paying groups in period t,

$b(t)$ = vector of tax subsidy-receiving groups in period t, and

$R(t)$ = matrix of participation rates in period t.

We then have the relations

(1) $a(t) = R(t)\,p(t)$ = vector of economically active population groups in period t, and

(2) $b(t) = \left(I-R(t)\right)p(t)$ = vector of economically inactive population groups in period t, where I represents the identity matrix.

Further, let $x_i(t)$ be the "tax" per person paid by the economically active members of the i^{th} tax-paying group in period t and let $y_j(t)$ be the "subsidy" given to the economically inactive members of the j^{th} subsidy-receiving group in period t. Then over a specified horizon $t = 1, \ldots, T_0$, we might require that "collections" be at least equal to "payments" for each time period as in, for example, the social security system. That is, we require

$$\sum_i x_i(t)a_i(t) > \sum_j y_j(t)b_j(t), \qquad t = 1, \ldots, T_0$$

or in matrix notation

$$(3) \quad x^T(t)a(t) > y^T(t)b(t), \qquad t = 1, \ldots, T_0$$

Substituting relationship (1) and (2) in (3), we obtain

$$x^T(t)R(t)p(t) > y^T(t)\big(I-R(t)\big)p(t), \qquad t = 1, \ldots, T_0$$

or equivalently

$$(4) \quad \big(x^T(t) + y^T(t)\big)R(t)p(t) > y^T(t)p(t), \qquad t = 1, \ldots, T_0$$

which defines a convex set in the non-negative variables $x_i(t)$, $y_j(t)$ and $r_{ij}(t)$ (the elements of R).

If p(t) were unknown, we might assume that the vector of demographic groups in any period is obtained from the corresponding vector in the previous period by means of a known transition matrix M invariant over time. Thus

$$p(t) = Mp(t-1) = \cdots = M^t p(0).$$

An example of such a matrix is given by the cohort survival matrix (see [5], [7], and [9]).

$$M = \begin{bmatrix} b_1 & b_2 & b_3 & \cdot & \cdot & \cdot & b_{n-1} & b_n \\ s_1 & & & & & & & \\ & s_2 & & & & & & \\ & & s_3 & & & & & \\ & & & \cdot & & & & \\ & & & & \cdot & & & \\ & & & & & \cdot & & \\ & & & & & & s_{n-1} & 0 \end{bmatrix}$$

where b_i and s_i are age specific birth and survival rates, respectively. In any given period t, b_i denotes the rate at which group i produces children who

will then become members of the first group in period t+1; and s_i denotes the percentage of group i that survives to become members of group i+1 in period t+1. Note that here we do not allow the members of any group to remain in that group from one period to the next.

In this case, projecting from time t=0 with the transition matrix M^t = MM...M (t times), we can rewrite (4) as

$$(4') \quad \left(x^T(t) + y^T(t)\right)R(t)M^t p(0) > y^T(t)M^t p(0), \quad t = 1, \ldots, T_0$$

As an indication of the further conditions which should be imposed on the vectors x(t) and y(t), we might require that the subsidies be at least equal to (and the taxes no greater than) some specified amounts,

$$(5) \quad y(t) > \overline{y}(t) \text{ and } x(t) < \overline{x}(t), \qquad t = 1, \ldots, T_0.$$

In addition, we might require that the participation rates be kept within prescribed bounds,

$$(6) \quad R(t) < \overline{R}^u(t) \text{ and } R(t) > \overline{R}^L(t), \qquad t=1, \ldots, T_0.$$

Equity might require that a cohort over its life cycle should receive benefits that are at least equal to the payments made by this cohort into the system. In order to illustrate how this kind of principle might be incorporated we choose a horizon that contains one whole cohort life cycle. We shall then impose this equity condition as a goal for a cohort that completes such a life cycle within the horizon.

Since upper bounds on payments to society as a whole are already provided by (3), our goal for the life cycle cohort will be to maximize its total lifetime benefits.

$$(7) \quad \sum_{t=1}^{\tau} y^T(t)\left(I-R(t)\right)\overline{M}^{t-1} e_1 \cdot e_1^T \cdot Mp(o)$$

where

$$\overline{M} = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ s_1 & & & & & \\ & s_2 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & s_{n-1} & 0 \end{bmatrix}, \quad e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

and τ denotes the life cycle period, $\tau < T_0$.

The expression (7) represents the life cycle benefits for the cohort starting at t=1. (Equity considerations can be developed for other cohorts by similar expressions but we shall not undertake this here.)

To determine our R(t) values endogenously we can begin by postulating an aggregate, economy-wide scenario of, say, GNP growth over time, which is exogenous to our system. This is done only for simplicity since explicit treatment of the dynamic interactions between participation rates and GNP and between tax rates, investment and GNP would unduly complicate the development we are undertaking here. With current estimates of productivity and population demographics we can project "reasonable" target levels for the R(t) values. We can then provide a clear picture of a simplified use of the mechanism of goal focusing as part of an aggregative analytical method for obtaining participation rates corresponding to the time dependent scenarios we wish to investigate.

A reasonable objective for our choice of tax rates might be to minimize the maximum tax rate over the horizon. In addition we have the goal of maximization of the life cycle benefits to the cohort starting at t=1[1] plus the

[1]The pay-as-you-go conditions (4'), will ensure an upper bound to this maximum.

goals associated with goal focusing determination of the participation rates. For simplicity we shall denote the latter by $G(t, x(t), y(t))$. Thus, through the goal focusing mechanism, the $R(t)$ will be determined in an endogenous manner that is dependent on the $x(t)$ and $y(t)$ as determined from, say, regression estimates.

We now proceed to formulate our goal focusing model in the following time dependent fashion. Let

$G(t, x(t), y(t))$ = matrix of "goal" participation rates in period t

$G^{+}(t), G^{-}(t)$ = matrices of positive and negative deviations, respectively, from $G(t, x(t), y(t))$.

$W^{+}(t), W^{-}(t)$ = matrices of weights associated with $G^{+}(t)$ and $G^{-}(t)$, respectively.

Then our goal focusing model can be written as

$$(8) \quad \text{Min } \lambda - W_0 \sum_{t=1}^{T} y^T(t)\left(I-R(t)\right)\overline{M}^{t-1} e_1 \cdot e_1^T M p(0) + \sum_{t=1}^{T_0} e^T W^{+}(t) G^{+}(t) e + \sum_{t=1}^{T_0} e^T W^{-}(t) G^{-}(t) e$$

subject to

$$\begin{array}{lll}
\lambda e^T - x^T(t) & \geq 0, & t=1, \ldots, T_0 \\
y(t) & \geq \overline{y}(t), & t=1, \ldots, T_0 \\
-x(t) & \geq -\overline{x}(t), & t=1, \ldots, T_0 \\
-R(t) & \geq -\overline{R}^u(t), & t=1, \ldots, T_0 \\
R(t) & \geq \overline{R}^L(t), & t=1, \ldots, T_0 \\
(x^T(t) + y^T(t)R(t)p(t) - y^T(t)p(t) & \geq 0, & t=1, \ldots, T_0 \\
R(t) \quad -G^{+}(t) + G^{-}(t) & = G\left(t,x(t),y(t)\right), & t=1, \ldots, T_0 \\
x(t),\ y(t),\ R(t),\ G^{+}(t),\ G^{-}(t) & \geq 0, & t=1, \ldots, T_0
\end{array}$$

Before proceeding to an illustrative numerical example we show that we can linearize the above nonlinear programming problem by making the following approximations.

We first introduce the new variable $z(t)$ defined by

$$(9) \quad z(t) \equiv x(t) + y(t)$$

and rewrite (4) as

$$(10) \quad z^T(t)R(t)p(t) - y^T(t)p(t) > 0, \qquad t=1, \ldots, T_0.$$

We can then approximate the nonlinear inequality (10) by

$$(11) \quad z^{\circ T}(t)R^{\circ}(t)p(t) + \left[\left(z^T(t) - z^{\circ T}(t)\right)R^{\circ}(t) + z^{\circ T}(t)\left(R(t) - R^{\circ}(t)\right)\right]p(t)$$
$$- y^T(t)p(t) > 0, \qquad t=1, \ldots, T_0$$

where $z^{\circ}(t)$ and $R^{\circ}(t)$ are suitably chosen reference points for $z(t)$ and $R(t)$. Inequality (11) then simplifies to

$$(11') \quad z^T(t)R^{\circ}(t)p(t) + z^{\circ T}(t)R(t)p(t) - y^T(t)p(t) > z^{\circ T}(t)R^{\circ}(t)p(t),$$
$$t=1, \ldots, T_0.$$

By a similar approximation our objective function in (8) may be linearized to

$$\text{Min}\lambda - W_0 \sum_{t=1}^{T}\left[y^T(t)\left(I-R^{\circ}(t)\right) - y^{\circ T}(t)R(t) + y^{\circ T}(t)R^{\circ}(t)\right]M^{t-1}e_1 \cdot e_1^T Mp(0)$$

$$+ \sum_{t=1}^{T_0} e^T W^+(t)G^+(t)e + \sum_{t=1}^{T_0} e^T W^-(t)G^-(t)e.$$

Finally, we take the "goal" participation rates $G(t, x(t), y(t))$ to be linear in $x(t)$ and $y(t)$ with variations around $R(0)$, the participation rates at time $t = 0$. Thus we have

$$(12)\quad e^T G(t, x(t), y(t)) = e^T R(0) - x^T(t)A - y^T(t)B.$$

Substituting (9), (11') and (12) in (8), a linear approximation to our goal focusing model can then be written as

$$(13)\quad \text{Min}\lambda - W_0 \sum_{t=1}^{\tau} \left[y^T(t)\left(I-R^*(t)\right) - y^{*T}(t)R(t) + y^{*T}(t)R^*(t)\right]\overline{M}^{t-1} e_1 \cdot e_1^T Mp(0)$$

$$+ \sum_{t=1}^{T_0} e^T W^+(t)G^+(t)e + \sum_{t=1}^{T_0} e^T W^-(t)G^-(t)e$$

subject to

$$\begin{array}{lll}
\lambda e^T - x^T(t) & \geq 0, & t=1, \ldots, T_0 \\
y(t) & \geq \overline{y}(t), & t=1, \ldots, T_0 \\
-x(t) & \geq -\overline{x}(t), & t=1, \ldots, T_0 \\
-R(t) & \geq -\overline{R}^u(t), & t=1, \ldots, T_0 \\
R(t) & \geq \overline{R}^L(t), & t=1, \ldots, T_0 \\
x(t)+y(t) \quad -z(t) & = 0 & t=1, \ldots, T_0 \\
-y^T(t)p(t)+z^{*T}(t)R(t)p(t)+z^T(t)R^*(t)p(t) & \geq z^{*T}(t)R^*(t)p(t), & \\
 & & t=1, \ldots, T_0 \\
x^T(t)A+y^T(t)B+e^T R(t) \quad -e^T G^+(t)+e^T G^-(t) & = e^T R(0) & t=1, \ldots, T_0 \\
x(t), y(t), z(t), R(t), G^+(t), G^-(t) & \geq 0, & t=1, \ldots, T_0
\end{array}$$

<u>Illustrative Example</u>

To illustrate our procedure consider an economy in which the population is grouped by age into four categories as shown below.

Group	Age Yrs.	Population p(0)
1	0-19	70.5 m.
2	20-39	70.0
3	40-59	46.1
4	60+	35.5

We assume that births occur only within the second group, i.e., among those between 20 and 39 years of age. The yearly birth rate for that group is taken to be 3 million per 70 million or 0.043, so that the birth rate is 0.86 over the twenty year period.

The matrix of cohort survival rates is given below and is to be interpreted in the following way. In any period t, the entry in row 1 column 2 is the birth rate for the second group, all other birth rates are zero; while the entry in row 3 column 2 indicates that 0.97 of the population of group 2 will survive to become the members of group 3 in the subsequent period. For completeness we assign a survival rate of ϵ (the entry in row 4 column 4) to the oldest age group but assume it is insignificant for our analysis here.

Cohort Survival Matrix

From Group

$$
\begin{array}{c c}
\text{To Group} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0.86 & 0 & 0 \\ 0.97 & 0 & 0 & 0 \\ 0 & 0.97 & 0 & 0 \\ 0 & 0 & 0.87 & \epsilon \end{bmatrix}
\end{array}
$$

We shall take our time horizon to be four periods each of twenty years duration so that we may follow the youngest cohort over its entire life cycle.

Participation rates at time t = 0 are given below.

<u>Initial Participation Rates R(0)</u>

$$
\begin{array}{c c}
 & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0.14 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0 \\ 0 & 0 & 0.70 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix}
\end{array}
$$

We assume that the participation rate for any group is influenced by the levels of tax and subsidy for that group but we here exclude any possible cross-group effects of taxes and subsidies, so that the matrices A and B of (12) and (13) have the particular (diagonal) forms

$$
A = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{bmatrix}, \qquad B = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{44} \end{bmatrix}
$$

We estimate that

$$
\begin{aligned}
a_{11} &= 0.02 \times 10^{-6}; & b_{11} &= 0.465 \times 10^{-6} \\
a_{22} &= 0.115 \times 10^{-6}; & b_{22} &= 1.12 \times 10^{-6} \\
a_{33} &= 0.67 \times 10^{-6}; & b_{33} &= 2.7 \times 10^{-6} \\
a_{44} &= 3.9 \times 10^{-6}; & b_{44} &= 6.5 \times 10^{-6}
\end{aligned}
$$

The model was run with different sets of values for the bounds on taxes, subsidies and participation rates, and for the objective function weights

where, for simplicity, the values were held constant over time. For illustrative purposes, we report the results of the (typical) case shown below.

Participation Rates

Lower Bound | Upper Bound

$$\overline{R}^L(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \forall t; \quad \overline{R}^u(t) = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0.4 \end{bmatrix}, \forall t.$$

Minimum levels for subsidies were taken to be the same for all groups. We assumed a value of $5,200 per year which is equivalent to $0.104m. over a twenty year period.

Lower Bounds on Subsidies $m

$$\overline{y}(t) = \begin{bmatrix} 0.104 \\ 0.104 \\ 0.104 \\ 0.104 \end{bmatrix}, \forall t.$$

Maximum tax levels per year and their equivalents over a twenty year period were taken to be as follows:

group 1: $ 4,000/yr.

group 2: 6,666

group 3: 10,000

group 4: 5,000

Upper Bounds on Taxes $m.

$$\bar{x}(t) = \begin{bmatrix} 0.08 \\ 0.13332 \\ 0.2 \\ 0.1 \end{bmatrix}, \forall t.$$

The weights $W^+(t)$ and $W^-(t)$ were taken to be unity for all groups in all time periods, while W_0, the weight on maximizing life time benefits for the single cohort, was taken to be 0.1.

Results

The results for our illustrative example are given below, but we note here the following observations. As required, total tax collections exceed subsidy payments; however, the differences range between $1.2b and $48.4b annually. We expect appropriate selection of time dependent weights and relative weights would bring collections and payments closer in line.

As expected, lifetime benefits to our single cohort greatly exceed tax receipts from the cohort, by a ratio of almost 2:1 in our example. This may occur because our model allows each economically inactive member of society to receive a subsidy, which is not the reality for the youngest group. Based on our results, if we were to remove the subsidy to our illustrative cohort in period one, the taxes paid by that cohort over its life cycle would exceed the benefits it receives. However, if we employ this procedure for the youngest group in each period, the gap between collections and payments for society as a whole widens considerably. We expect such discrepancies would be diminished significantly by including in the objective function life-time-benefits for each cohort rather than for only our illustrative one. This, of course, is the appropriate situation for any real application, and also provides valuable

Illustrative Example Results

Group 1		Period 1	Period 2	Period 3	Period 4
Population	M	60.200000	58.811100	50.218840	49.060220
Participation Rate		.049871	.089520	.089520	.089520
Lower Bound		0.000000	0.000000	0.000000	0.000000
Upper Bound		.200000	.200000	.200000	.200000
Participants	M	3.002234	5.264770	4.495591	4.391871
Tax/Year	$	4000	4000	4000	4000
Uppper Bound	$	4000	4000	4000	4000
Collections/Year	$B	12.008937	21.059079	17.982362	17.567483
Collections/Period	$T	.240179	.421182	.359647	.351350
Non Participants	M	57.197766	53.546330	45.723249	44.668349
Subsidy/Year	$	9418	5200	5200	5200
Lower Bound	$	5200	5200	5200	5200
Payments/Year	$B	538.688558	278.440918	237.760897	232.275414
Payments/Period	$T	10.773771	5.568818	4.755218	4.645508

Group 2					
Population	M	68.385000	58.394000	57.046767	48.712275
Participation Rate		.800000	.617522	.800000	.670586
Lower Bound		.600000	.600000	.600000	.600000
Upper Bound		.800000	.800000	.800000	.800000
Participants	M	54.708000	36.059580	45.637414	32.665770
Tax/Year	$	6666	6666	6666	6666
Uppper Bound	$	6666	6666	6666	6666
Collections/Year	$B	364.683528	240.373158	304.218999	217.570020
Collections/Period	$T	7.293671	4.807463	6.084380	4.355000
Non Participants	M	13.677000	22.334420	11.409353	16.046505
Subsidy/Year	$	5200	5200	5200	5200
Lower Bound	$	5200	5200	5200	5200
Payments/Year	$B	71.120400	116.138986	59.328638	83.441828
Payments/Period	$T	1.422408	2.322780	1.186573	1.668837

Illustrative Example Results

Group 3		Period 1	Period 2	Period 3	Period 4
Population	M	67.900000	66.333450	56.642180	55.335364
Participation Rate		.800000	.800000	.600000	.800000
Lower Bound		.600000	.600000	.600000	.600000
Upper Bound		.800000	.800000	.800000	.800000
Participants	M	54.320000	53.066760	33.985308	44.268291
Tax/Year	$	10000	10000	10000	10000
Uppper Bound	$	10000	10000	10000	10000
Collections/Year	$B	543.200000	530.667600	339.853080	442.682912
Collections/Period	$T	10.864000	10.613352	6.797062	8.853658
Non Participants	M	13.580000	13.266690	22.656872	11.067073
Subsidy/Year	$	5200	5200	5200	5200
Lower Bound	$	5200	5200	5200	5200
Payments/Year	$B	70.616000	68.986788	117.815734	57.548779
Payments/Period	$T	1.412320	1.379736	2.356315	1.150976
Group 4					
Population	M	40.107000	59.073000	57.710101	49.278697
Participation Rate		0.000000	.059948	.180504	0.000000
Lower Bound		0.000000	0.000000	0.000000	0.000000
Upper Bound		.400000	.400000	.400000	.400000
Participants	M	0.000000	3.541308	10.416904	0.000000
Tax/Year	$	0	0	0	0
Uppper Bound	$	5000	5000	5000	5000
Collections/Year	$B	0.000000	0.000000	0.000000	0.000000
Collections/Period	$T	0.000000	0.000000	0.000000	0.000000
Non Participants	M	40.107000	55.531692	47.293197	49.278697
Subsidy/Year	$	5200	5200	5200	5200
Lower Bound	$	5200	5200	5200	5200
Payments/Year	$B	208.556400	288.764797	245.924626	256.249222
Payments/Period	$T	4.171128	5.775296	4.918493	5.124984
Total Collections/Year	$T	.919892	.792100	.662054	.678000
Total Payments/Year	$T	.888981	.752331	.660830	.629515
Total Collections/ Period	$T	18.397849	15.841997	13.241089	13.560008
Total Payments/Period	$T	17.779627	15.046630	13.216598	12.590305
Total Societal Collections	$T	61.040943			
Total Societal Payments	$T	58.633160			
Total Cohort Collections	$T	11.844703			
Total Cohort Payments	$T	20.577850			

insight into the possible effects of cross-subsidization, between groups and across time.

The preceding comments also go far in explaining the following results. With one exception, that of group one in the first period, subsidies were consistently at their lower bounds; on the other hand, with the exception of the oldest group in each period where taxes were zero, taxes were at their upper bounds. Participation rates for the youngest and oldest groups were consistently close to their lower bounds of zero, while for the middle two groups the rates either were at their upper bounds or close to their lower bounds depending on the period considered.

The results from our illustrative example provide some preliminary indications of the scope of the model and the kinds of insights to be obtained. Clearly, the experience gained here suggests further developments, along the lines outlined in the next section. Subsequently, more realistic applications can be made.

Extensions and Modifications

In current work, the basic model is being extended and modified in several directions and more comprehensive applications are being analyzed.

The procedure for determining the vector of demographic groups in any period will be refined. This will involve elaboration of the transition matrix M. In particular, we can allow for members of any group to remain in that group from one period to the next, although this is contingent on how narrowly we define a group and a time period for any given application.

Explicit treatment of the dynamic interactions between participation rates and GNP and between tax rates, investment and GNP need be made. That is, the model should incorporate the effects of economic growth on the retirement system.

There exist two negative connections between the levels of transfer payments and economic growth. One is that the taxes raised to pay for the transfer payments may reduce the investment in fixed capital by more than the expenditures made by the benefit recipients contribute to stimulate investment through growth in the demand for output. The second channel by which transfer payments adversely affect economic growth is through their impact on labor force participation of older population groups. Thus, since participation rates are determined endogenously in our model through the goal focusing mechanism, we need improved estimates of the relationship between participation rates and the levels of taxes and subsidies. In this context, we might require that taxes and subsidies for certain groups stand in particular relationship to those for other groups; and that tax and subsidy levels be allowed to increase only at certain minimum and maximum rates over time.

Alternative criteria of stability can be explored rather than simply requiring "collections" to at least equal "payments" in each time period. We would require this to be so over a given time horizon, but there is scope to relax this condition for particular time periods and analyze the trade-off involved.

Similar analyses are possible with alternative criteria of equity. The present model requires that a cohort over its life cycle should receive benefits that are at least equal to the payments made by this cohort into the system. A more comprehensive example may extend this requirement to all groups, but the general approach also allows for modification of this condition for particular groups over specified time horizons; that is, it affords analysis of the trade-off involved in cross-subsidization. These equity considerations are built in as part of the goal functional along with other

reasonable objectives such as minimizing the maximum tax rate or maximizing the minimum subsidy rate over the time horizon.

Summary

This paper has proposed an operationally implementable method for the analysis of intergenerational transfers of income employing the new approach of goal focusing. This development improves on existing models through its capability for analysis of the numerous trade-offs and relative possibilities inherent in an income transfer program. Its linear programming equivalents afford computational efficiency and easy access to sensitivity analyses.

A basic theoretical model has been presented and an illustrative example has demonstrated the general procedure involved and the kinds of insights to be obtained. Extensions in several directions, some currently underway, have been described.

Such models will provide considerable assistance in obtaining substantive knowledge of the prospective magnitudes of the trade-off involved and their behavior within a set of plausible trends over the next ten to thirty years. Such information is central in striving to attain simultaneously the socially desirable goals of maintaining or improving the standards of living of both the working and retired populations and of maintaining an equitable and stable intergenerational transfer system.

REFERENCES

1. A. Charnes and W. W. Cooper, Management Models and Industrial Application of Linear Programming, Vols. I and II, Wiley, New York, 1961.

2. A. Charnes and W. W. Cooper, "Goal Programming and Multiple Objective Optimizations," European Journal of Operational Research, 1, No.1, 1977.

3. A. Charnes, W. W. Cooper, and E. Rhodes, "Measuring the Efficiency of Decision Making Units," European Journal of Operational Research, 2, No.6, 1978.

4. A. Charnes, W. W. Cooper, J. J. Rousseau, and A. Schinnar, "A Goal Focusing Extension for Intergenerational Transfers of Income," in Production of Well-Being, N. E. Terleckyj, ed., Washington, D.C., National Planning Association.

5. Keyfitz, N., Introduction to the Mathematics of Population, Addison-Wesley, Reading, Mass., 1968.

6. D. Levy and N. E. Terleckyj, "Intergenerational Transfers of Economic Well-Being," in Production of Well-Being, N. E. Terleckyj, ed., Washington, D.C., National Planning Association.

7. Pollard, J. H., Models for Human Populations, Cambridge University Press, 1973.

8. Production of Well-Being, N. E. Terleckyj, ed., Washington, D.C., National Planning Association (forthcoming).

9. Rogers, A., Introduction to Multi Regional Mathematical Demography, Wiley, New York, 1975.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER CCS 466	2. GOVT ACCESSION NO. AD-A138 137	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Goal Focusing Approach to Analysis of Intergenerational Transfers of Income: Theoretical Development and Preliminary Results		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) A. Charnes, W.W. Cooper. J. Rousseau, A. Schinnar and N.E. Terleckyi		8. CONTRACT OR GRANT NUMBER(s) N00014-82-K-0295
9. PERFORMING ORGANIZATION NAME AND ADDRESS Center for Cybernetic Studies The University of Texas at Austin Austin, TX 78712		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research (Code 434) Washington, DC		12. REPORT DATE August 1983 13. NUMBER OF PAGES 25
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) This report has been approved for public release and sale; its distribution is unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Goal focusing Intergenerational income transfers Goal programming Pareto efficiency		

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Existing analytical models of intergenerational transfers of income developed by economists deal with the criterion of equity of retirement systems but not with their stability. These models frequently assume a steady growth in population. However, present and projected population dynamics are characterized by very large changes in the age structure of the population. Governmental and private projection models used for analysis of government policies for social security and other age-determined transfer programs do rely on the cur-

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

rent population projections, but they assume independence of future economic growth from the retirement system.

Moreover, none of these models are formulated in a way that makes them operational for studying the numerous dynamic trade-offs and relative possibilities inherent in income transfer programs; for example, the trade-off between taxes and subsidies, between participation rates and income for different groups over time, and the possible effects of cross-subsidization among groups of different compositions and sizes. Such information is central in striving to attain simultaneously the socially desirable goals of maintaining or improving the standards of living of both the working and retired populations and of maintaining an equitable and stable intergenerational transfer system.

This paper proposes such an operationally implementable method of analysis employing the new approach of goal focusing, derived from ideas of goal programming and efficient point analysis. A basic theoretical model is developed and illustrated by numerical example, and extensions in several directions are described. Such models will provide considerable assistance in obtaining substantive knowledge of the prospective magnitudes of the trade-offs involved and their behavior within a set of plausible trends over the next ten to thirty years.

END

FILMED

3-84

DTIC